Successive Interference Cancellation With Feedback for Random Access Networks

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***Abstract*— We consider a random access network in which K nodes wish to send independent packets to an access point (AP). In this letter, a novel method of feedback construction and an adaptive retransmission protocol of collided packets are proposed, which enable efficient successive interference cancellation at the AP. We show that the optimal throughput efficiency of one is achievable by only exploiting $\log K$ bits of feedback from the AP to K nodes, while the maximum throughput efficiency of slotted ALOHA is known to be $e^{-1} \approx 0.37$ for large K. Note that the proposed technique achieves the optimal throughput efficiency for any finite K, while the conventional techniques asymptotically achieve the optimal throughput efficiency only when K tends to infinity.**

***Index Terms*— Random access, slotted ALOHA, successive interference cancellation, feedback, throughput efficiency, machine-to-machine (M2M) communications, Internet-of-Things (IoT).**

I. INTRODUCTION

RECENTLY, random access (RA) techniques have received much interest as various machine-to-machine (M2M) communications emerged, supporting a massive number of uncoordinated nodes in a network. The RA is also expected to play an important role in realizing internet-of-things (IoT) which connects a massive number of machine nodes in a wide range of applications such as smart metering, surveillance, security, infrastructure management, city automation, and e-health. Among various RA techniques, slotted ALOHA is a simple and widely used scheme [1]. The maximum throughput efficiency of slotted ALOHA is known to be $e^{-1} \approx 0.37$ for large K, where K denotes the number of nodes in a network. In the slotted ALOHA protocol, collided packets at an access point (AP) are simply discarded.

After the introduction of slotted ALOHA, there have been many studies to enhance the throughput efficiency by utilizing collided packets at the AP for decoding instead of discarding them [2]–[5]. In particular, successive interference cancellation (SIC) was combined with diversity transmission of data bursts in [2], which improves the throughput efficiency approximately to 0.55. In [3]–[5], several SIC techniques have been proposed by allowing a variable repetition rate for each packet, based on the density evolution analysis of low-density parity-check (LDPC) codes. In [5], it was shown that such a technique asymptotically achieves the optimal throughput efficiency of one as K tends to infinity.

The aforementioned schemes in [1]–[5] assumed feedback from the AP to nodes in order to avoid packet loss due to collisions. On the other hand, there have been studies on RA protocols without feedback [6]–[9]. In particular, a feedback-free RA technique was shown to asymptotically achieve the optimal throughput efficiency of one with an arbitrarily small probability of packet loss as K tends to infinity [9]. We note that the optimal throughput efficiency of one has been shown to be achievable only asymptotically as K tends to infinity both with or without feedback [3]–[5], [9].

In this letter, we show that the optimal throughput efficiency of one is achievable with $\log K$ bits of feedback for any finite K. Furthermore, we establish the optimal throughput efficiency by a simple adaptive retransmission protocol based on the conventional point-to-point physical layer coding technique without modifying it or introducing more complicated coding techniques.

II. PROBLEM FORMULATION

In this section, we present the network model and the performance metric. For two integers a and b, $[a : b]$ denotes the set $\{a, a+1, \cdots, b\}$ throughout this letter.

A. Network Model

Consider a (distributed) RAN in which K nodes wish to send independent packets to a single AP. In this letter, we focus on the uplink scenario. We assume that the transmission time is equally divided by slots, as in the slotted ALOHA. In each slot, each node transmits a new packet to the AP with probability $\alpha \in [0, 1]$. Then, on average, each node transmits a new packet with rate α packets/slot. The received signal vector at the AP at slot t is given by $\mathbf{y}_t = \sum_{i=1}^{K} h_i \mathbf{x}_{i,t} + \mathbf{z}_t$, where h_i is the wireless channel coefficient from node i to the AP, $\mathbf{x}_{i,t}$ is the transmitted signal vector from node i, and $\mathbf{z}_t$ is the additive Gaussian noise vector at the AP, at slot t. The vector length is determined by the packet length. For convenience, we assume that all packets from nodes have the same length each other. We assume that wireless channel coefficients remain unchanged throughout the communication.

In this letter, we assume a simple feedback mechanism from the AP. The AP broadcasts an acknowledgement (ACK) packet if a single packet arrives and the packet is successfully decoded. However, if more than one node simultaneously send their packets in the RAN, then packet collision will occur. When a collision happens, two different feedback mechanisms can be utilized. First, the AP may ignore the collided packets.

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Then, all nodes in the RAN implicitly realize that the packets are not successfully decoded at the AP after a certain time duration. This feedback mechanism is known as positive feedback because the AP only sends ACK packets to the nodes. Second, on the other hand, the AP can broadcast some information about the collision, e.g., the indices of nodes experiencing the collision, to all nodes in the RAN. The second feedback mechanism has been shown to be feasible through the unequal packet protection technique in which the identifier of colliding packets is sent via the most robust modulation and coding scheme (MCS), while data is sent via a proper MCS according to channel gains [10]. In this letter, we consider the second feedback mechanism for the case of collision and, therefore, assume that the AP knows the set of collided nodes in each slot and broadcasts N-bit feedback information after receiving multiple packets from nodes, which will be determined in the next section. We will demonstrate that SIC combined with a novel retransmission protocol based on this feedback information can drastically improve the throughput efficiency of RANs. For describing our main result, we also define the normalized throughput and throughput feasibility for RANs with N-bit feedback in the following subsection.

B. Normalized Throughput in RANs

In this letter, the amount of information bits conveyed by a packet at node $i \in [1:K]$ is set as the interference-free channel capacity from node i to the AP, i.e., $WD\log_2(1+|h_i|^2P_i)$ bits/packet, where W denotes the bandwidth, D denotes the duration of each slot, and P_i denotes the transmit power of node i. Let R_i denote the average throughput of node i. Since each node transmits a new packet with rate α packets/slot on average, if there is no collision, R_i will become $\alpha WD\log_2(1+|h_i|^2P_i)$ bits/slot. However, packet collisions may decrease the average throughput of each node in the RAN. In practical RANs, nodes are located in different distances from the AP and, as a result, the channel gains $\{|h_i|^2\}_{i=1}^K$ and the resultant average throughputs $\{R_i\}_{i=1}^K$ are also different from each other. In order to provide a unified and simplified framework for analyzing throughput in the RAN, we define the normalized throughput as $T_i \triangleq R_i/(WD\log_2(1+|h_i|^2P_i)) \in [0,\alpha]$, which is interpreted as the number of successfully decodable packets per each slot for node i. The formal definition of the normalized throughput is given as follows:

Definition 1 (Normalized Throughput): In this letter, a normalized throughput tuple $(T_1,\cdots,T_K)$ is said to be achievable for the RAN defined in Section II-A if there exists a transmission protocol such that the AP is able to successfully decode packets transmitted from node i with a rate of $T_i \in [0,\alpha]$ (packets/slot) for all $i \in [1:K]$. ♢

Hence, if $(T_1,\cdots,T_K)$ is achievable, node i is able to transmit its packets with the average data rate of $T_iWD\log_2(1+|h_i|^2P_i)$ bits/slot. Unless otherwise stated, throughput means the normalized throughput in the rest of this letter.

Definition 2 (Feasibility of α): In this letter, α is said to be feasible for the RAN defined in Section II-A if there exists an achievable $(T_1,\cdots,T_K)$ satisfying that $T_i \geq \alpha-\epsilon$ for all $i \in [1:K]$, where $\epsilon > 0$ is an arbitrarily small constant. ♢

III. SIC With Feedback

In this section, we present our main results on the achievable throughput and construct the achievability scheme. Let $G_i \in [0,1]$ (packets/slot) denote the transmission rate of packets at node i including retransmissions due to collisions. Obviously, $T_i \leq G_i$ since T_i only counts successfully decoded packets at the AP. Notice that the relation between $\{G_i\}_{i=1}^K$ and $\{T_i\}_{i=1}^K$ depends on a transmission protocol. Now, we are ready to present the main theorem.

Theorem 1: For the RAN with $N=\log K$ bits of feedback, there exists a transmission protocol with $G_i\prod_{j=1}^{i-1}(1-G_j) \leq \alpha$ for $i \in [1:K]$ that achieves the following throughput tuple:

$$\left\{T_i = G_i\prod_{j=1}^{i-1}(1-G_j)\right\}_{i=1}^{K}. \tag{1}$$

Proof: We propose a transmission protocol utilizing feedback information of $\log K$ bits in Section III-A, illustrate SIC-based decoding at the AP in Section III-B, and analyze its throughput tuple $(T_1,\cdots,T_K)$ in Section III-C. ■

Remark 1: For the general case where node i transmits a new packet with probability α_i, one can check by closer inspection of our proposed protocol that Theorem 1 holds by replacing α by α_i, i.e., the throughput tuple can be evaluated from (1) by substituting $G_1,\cdots,G_K$ that satisfy $G_i\prod_{j=1}^{i-1}(1-G_j) \leq \alpha_i$ for $i \in [1:K]$. ♢

The following corollary states that $\alpha = 1/K$ is feasible from Theorem 1.

Corollary 1: For the RAN with $N = \log K$ bits of feedback, $\alpha = 1/K$ is feasible.

Proof: Suppose that $\alpha = 1/K$. Fix arbitrary $\epsilon > 0$. For $\epsilon' > 0$, let $G_i = 1/(K-i+1)-\epsilon'$ for $i \in [1:K]$. Then, from Theorem 1, $T_i \geq 1/K-\epsilon$ for $i \in [1:K]$ is achievable by choosing ϵ' sufficiently small. Then, from Definition 2, $\alpha = 1/K$ is feasible. ■

Recall that the amount of information bits conveyed by each packet is set as the interference-free channel capacity. Therefore, the maximum feasible α is obviously upper bounded by $1/K$, meaning that the AP is able to decode at most one packet at each slot. Corollary 1 demonstrates that $\alpha = 1/K$ is indeed achievable for any finite K by incorporating $\log K$ bits of feedback from the AP to the K nodes in the RAN. For the rest of this section, we prove Theorem 1.

A. Transmission Protocol

In this subsection, we propose a transmission protocol utilizing feedback information of $\log K$ bits. For the proposed protocol, each node basically adopts slotted ALOHA and retransmits collided packets with random backoffs. Specifically each node sets a backoff time among $[1:B]$ uniformly at random and then retransmits a collided packet with the selected backoff time delay. Here, B is the maximum backoff time, which is a predetermined positive integer value. For convenience, we simply mention that 'set a random backoff' to describe the above procedure.

At each slot, each node may have a new packet and/or retransmission packets whose backoff counters expires. The following protocol describes the packet transmission (including retransmission) at slot t:

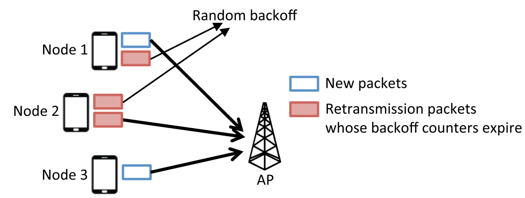

Fig. 1. Packet transmission example at slot t when $K = 3$.

- If a new packet arrives, node i transmits the new packet. For the retransmission packets whose backoff counters expire, reset their backoffs.
- If there is no new packet, node i transmits one of the retransmission packets whose backoff counters expire and reset their backoffs for the rest of the retransmission packets.
- If there is neither new packet nor retransmission packet whose backoff counter expires, node i does not transmit.

Now consider the random backoff procedure of the collided packets at the end of slot t. Notice that the retransmission procedure works packet basis instead of user basis. Recall that the AP broadcasts $\log K$ bits of feedback to K nodes at the end of slot t and each node will adjust its transmission based on such feedback information. In the following, we first state how to construct $\log K$ bits of feedback. Let $\mathcal{C}_t \subseteq [1 : K]$ denote the set of collided nodes transmitted at slot t. Note that $\mathcal{C}_t = \emptyset$ if a single node transmits at slot t. The feedback information F_t from the AP to the nodes at the end of slot t is constructed as $F_t = 0$ if $\mathcal{C}_t = \emptyset$ and otherwise $F_t = \min(\mathcal{C}_t)$. After receiving F_t from the AP, node i who attempted transmission in slot t operates as follows at the end of slot t:

- If $i \neq F_t$, node i sets a random backoff for the retransmission of the packet transmitted in slot t.
- If $i = F_t$, node i does not retransmit the packet transmitted in slot t.

Unlike the conventional random access protocols, all nodes in $\mathcal{C}_t$ except the node indexed by $\min(\mathcal{C}_t)$ retransmit their packets for the proposed protocol. Fig. 1 illustrates an example of the proposed packet transmission at slot t when $K = 3$. In this example, node 1 has a new packet and one retransmission packet (whose backoff counter expires) so that it transmits the new packet and resets its backoff for the retransmission packet. Node 2 has two retransmission packets so that it transmits one of them randomly and resets its backoff for the other retransmission packet. Node 3 has only a new packet and thus transmits it. Since nodes 1, 2, and 3 transmit their packets, $F_t = 1$ for this case. Hence, node 1 discards the collied packet and only nodes 2 and 3 reset their backoffs for the collided packets at the end of slot t.

B. SIC-Based Decoding

Now consider decoding at the AP. Denote the j-th new packet generated by node k as packet (k, j) and the set of packet indices transmitted in slot t as $\mathcal{A}_t$. Note that the same packet (k, j) can be transmitted for multiple times according to our transmission protocol. Now, let us describe the operation of the AP at slot t. First, if $F_t \neq 0$, the AP does not perform decoding and it stores the received signal vector $\mathbf{y}_t$ and the set $\mathcal{A}_t$. Next, if $F_t = 0$, the AP decodes according to the following procedures:

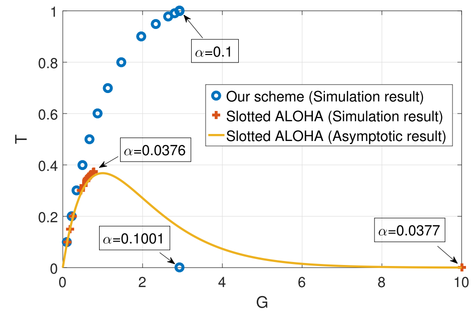

Fig. 2. For $K = 10$, numerical values of (G, T) by running our protocol for various values of α are plotted. For comparison, the asymptotic result $T = Ge^{-G}$ using slotted ALOHA as K tends to infinity and the simulation result by running slotted ALOHA for $K = 10$ are also plotted.

(i) Decode the packet transmitted in slot t, say packet (k, j), from $\mathbf{y}_t$ and let $\mathcal{A}_t = \emptyset$. For $t' \in [1 : t-1]$ such that $(k, j) \in \mathcal{A}_{t'}$, update $\mathcal{A}_{t'} = \mathcal{A}_{t'} \setminus \{(k, j)\}$ and $\mathbf{y}_{t'} = \mathbf{y}_{t'} - h_k \mathbf{x}_{k,t'}$.
(ii) For $t' \in [1 : t-1]$ such that $|\mathcal{A}_{t'}| = 1$, decode the packet in $\mathcal{A}_{t'}$, say packet (k', j'), from $\mathbf{y}_{t'} = h_{k'}\mathbf{x}_{k',t'} + z_{t'}$. Then, for $t'' \in [1 : t-1]$ such that $(k', j') \in \mathcal{A}_{t''}$, update $\mathcal{A}_{t''} = \mathcal{A}_{t''} \setminus \{(k', j')\}$ and $\mathbf{y}_{t''} = \mathbf{y}_{t''} - h_{k'}\mathbf{x}_{k',t''}$.
(iii) Repeat (ii) until there is no $t' \in [1 : t-1]$ such that $|\mathcal{A}_{t'}| = 1$.

We make two assumptions: 1) no error (an arbitrarily small probability of error) for decoding a single transmitted packet when there is no collision and 2) perfect interference cancellation for removing interference arisen from the decoded packet, which has been commonly assumed in the previous works, see [2]–[5]. Under such assumptions, it can be checked that all the packets are eventually decoded by the aforementioned decoding rule. To see this, first consider decoding of the packets transmitted from node K. From the transmission protocol in Section III-A, node K does not retransmit its packet when there is no concurrent transmission from the other $K-1$ nodes. Hence the AP is able to decode all transmitted packets from node K and cancel the signals due to the packets of node K from the corresponding received signal vectors. Now consider decoding of the packets transmitted from node $K-1$. Each packet can be transmitted from node $K-1$ for multiple times according to the retransmission protocol. Then focus on the received signal vector at the AP when the packet of interest is transmitted for the last time. For such lastly received signal vectors involving the packets of node $K-1$, there are two possible cases: 1) only node $K-1$ transmits and 2) only nodes $K-1$ and K transmit. For the first case, the AP can decode the packets of node $K-1$. For the second case, since the AP already cancelled the signals due to the packets of node K, the packets of node $K-1$ can be decoded. After that, the AP again cancels the signals due to the packets of node $K-1$. The decoding of the packets transmitted from nodes $K-2, \cdots, 1$ can be analyzed in a similar way.

C. Throughput Analysis

For tractable analysis, we assume that node $i \in [1 : K]$ transmits a packet, which can be a new packet or a

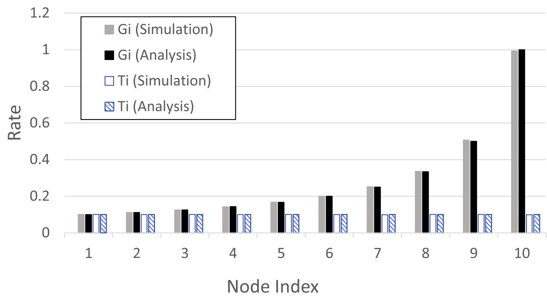

Fig. 3. Simulation and analytical results of $\{G_i\}_{i=1}^{10}$ and $\{T_i\}_{i=1}^{10}$ when $K = 10$ and $\alpha = 0.1$.

retransmission packet, at rate G_i in an independent and identically distributed (i.i.d.) manner over slots. Such an i.i.d. assumption is widely used in the literature, see e.g., [11], [12]. In Section IV, we show that such an i.i.d. assumption is well-matched with simulation results when the maximum backoff time B is large enough.

Denote by $\mathcal{E}_t(i)$ the event that node i transmits at slot t and denote by $\mathcal{E}_t^c(i)$ the complement of $\mathcal{E}_t(i)$. Then,

$$\begin{aligned} T_i &\overset{(a)}{=} \Pr(\mathcal{E}_t(i))\left(\Pr(F_t = i|\mathcal{E}_t(i)) + \Pr(F_t = 0|\mathcal{E}_t(i))\right) \\ &= \Pr(\mathcal{E}_t(i))\Bigg(\Bigg(\prod_{j=1}^{i-1}\Pr\left(\mathcal{E}_t^c(j)\right)\Bigg)\Bigg(1 - \prod_{j=i+1}^{K}\Pr(\mathcal{E}_t^c(j))\Bigg) \\ &\quad + \Bigg(\prod_{j=1}^{i-1}\Pr\left(\mathcal{E}_t^c(j)\right)\Bigg)\Bigg(\prod_{j=i+1}^{K}\Pr(\mathcal{E}_t^c(j))\Bigg)\Bigg) \\ &\overset{(b)}{=} G_i\prod_{j=1}^{i-1}(1 - G_j), \end{aligned} \tag{2}$$

where (a) follows because if node i transmits a packet at slot t and $F_t \in \{0, i\}$, the packet is not retransmitted and can be eventually decoded according to the decoding procedure in Section III-B and (b) follows from the i.i.d. assumption and the definition of G_i. Therefore, $\{T_i\}_{i=1}^{K}$ represented in (2) is achievable. In conclusion, Theorem 1 holds.

IV. Simulation Results

In this section, we evaluate the performance of the proposed scheme. In simulations, we assume $K = 10$, $B = 1000$, and a total of 10^7 time slots. Let $G = \sum_{i=1}^{K} G_i$ and $T = \sum_{i=1}^{K} T_i$ denote the total transmission rate (including retransmissions) and the total throughput, respectively. It is well-known that the slotted ALOHA asymptotically achieves $T = Ge^{-G}$ as K tends to infinity. In Fig. 2, for various values of α, numerical values of (G, T) obtained by running our protocol are plotted. For comparison, we also plot the asymptotic result $T = Ge^{-G}$ and simulation result when $K = 10$ for slotted ALOHA. We can see that our scheme can achieve up to $T = 1$, while the slotted ALOHA achieves at most $T = 0.376$. As seen in the figure, when $\alpha \geq 0.1001$ for our scheme and $\alpha \geq 0.0377$ for the slotted ALOHA, a collision always happens in steady state, which precludes decoding, and hence the total throughput T becomes zero.

TABLE I
Average Delay When $K = 10$ and $B = 100$

α	0.01	0.02	0.03	0.05	0.07	0.1
Our scheme	4.9	10.6	17.3	35.5	66.4	328
Slotted ALOHA	5.32	13.57	28.5			

For $K = 10$ and $\alpha = 0.1$, in Fig. 3, numerical values of $\{G_i\}_{i=1}^{10}$ and $\{T_i\}_{i=1}^{10}$ obtained by running our protocol are compared with $G_i = \frac{1}{10-i+1}$ and $T_i = \frac{1}{10}$ for $i \in [1 : 10]$, analyzed in Corollary 1. As shown in the figure, the simulated and analytical values coincide well, demonstrating that the i.i.d. assumption in the analysis is valid. Note that T_i's are the same because each node transmits a new packet with the same rate, while G_i increases in i since a node with larger index retransmits more according to our protocol.

Table I compares the delay performance of our scheme with slotted ALOHA when $K = 10$ and $B = 100$. For the slotted ALOHA, the maximum achievable α is known as $\frac{e^{-1}}{K}$ for large K. As seen in the table, our scheme provides not only a general throughput–delay trade-off available up to $\alpha = \frac{1}{K}$ but it also improves the delay performance than the slotted ALOHA because of the reduced number of retransmissions.

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